Module 1: Ratios and Proportional Relationships

Enduring Understandings:

Students will understand that ...

• A ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percents.

• A ratio or a rate expresses the relationship between two quantities. Ratio and rate language is used to describe a relationship between two quantities

(including unit rates.)

• A rate is a type of ratio that represents a measure, quantity, or frequency, typically one measured against a different type of measure, quantity, or frequency.

• Ratio and rate reasoning can be applied to many different types of mathematical and real-life problems (rate and unit rate problems, scaling, unit pricing,

Essential Questions:

statistical analysis, etc.).

- When is it useful to be able to relate one quantity to another?
- How are ratios and rates similar and different?
- What is the connection between a ratio and a fraction?

Essential Skills:

Students will be able to ...

- Use ratio language to describe a ratio relationship between two quantities. (6.RP.1)
- Represent a ratio relationship between two quantities using manipulatives and/or pictures, symbols and real-life situations. (a to b, a:b, or a/b) (6.RP.1)
- Represent unit rate associated with ratios using visuals, charts, symbols, real-life situations and rate language. (6.RP.2)
- Use ratio and rate reasoning to solve real-world and mathematical problems. (6.RP.3c)
- Make and interpret tables of equivalent ratios. (6.RP.3)
- Plot pairs of values of the quantities being compared on the coordinate plane. (6.RP.3)
- Use multiple representations such as tape diagrams, double number line diagrams, or equations to solve rate and ratio problems. (6.RP.3d)
- Solve unit rate problems (including unit pricing and constant speed). (6.RP.3a,b)
- Solve percent problems, including finding a percent of a quantity as a rate per 100 and finding the whole, given the part and the percent. (6.RP.3a,b)
- Use variables to represent two quantities in a real-world problem that change in relationship to one another. (6.EE.9)

• Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. (6.EE.9)

• Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. (6.EE.9)

Standards: RP.1, RP.2, RP.3

Vocabulary:

percent, proportion, rate, ratio, rational number, tape diagram, unit rate, quantity

Instructional Resources:

http://ime.math.arizona.edu/progressions/

http://www.engageny.org/mathematics

http://www.isbe.net/

Eureka Math

NCTM: Illuminations

Achieve the Core Modules

Hunt Institute videos

Tasks:

TASK: ICE CREAM OR CAKE?

Suppose you survey all the students at a school to find out whether they like ice cream or cake better as a dessert, and you record your results in the contingency table below.

	ice cream	cake	totals	
boys	82	63	145	
girls	85	70	155	
totals	167	133	300	

a) What percentage of students at this school prefers ice cream over cake?

b) At this school, are those preferring ice cream more likely to be boys or girls?

c) At this school, are girls more likely to choose ice cream over cake than boys are?

TASK: RATIOS AND RATES

A ratio can be expressed three ways:

- Using the fraction bar as in $\frac{2}{3}$
- Using a colon symbol as in 2:3
- Using the word "to" as in 2 to 3.

Write each ratio using the other two ways:

- 1. The ratio of 3 inches to 20 feet.
- 2. The ratio of 26 students: 1 class
- 3. The ratio of $\frac{2 \text{ boys}}{3 \text{ girls}}$

When the denominator of a rate is 1, we call the rate a unit rate. We usually use the key word "per" or the division symbol (/)to indicate a unit rate. For example: If a student earns \$7.65 per hour, it is the same as \$7.65/hour, and means \$7.65 for every hour of work.

Find the unit rate for the following:

4. 120 eggs from 20 chickens

5. \$55 for 20 people

6. 250 miles in 4 hours

7.60 feet in 4 minutes

8. 48 books for 16 students

9. 56 children from 14 families

Unit rates can also be used to solve problems.

10. Which is the better deal: 8 ounces of shampoo for \$0.89 or 12 ounces for \$1.47

11. Which is the better deal: 3 cans of soda for \$1.27 or 5 cans of soda for \$1.79

12. Which is the better deal: 10 pounds of hamburger for \$4.99 or 5 pounds of hamburger for \$2.69

13. Which is traveling faster: Traveling 300 miles in 5 hours or traveling 250 miles in 4 hours

14. Which is traveling faster: Traveling 75 miles in 1 hour or traveling 280 miles in 3.5 hours

15. Which is traveling faster: Traveling 150 yards in 40 seconds or traveling 406 feet in 35 second

Assessments:

http://www.engageny.org/mathematics

Module 2: NUMBER SYSTEMS: Arithmetic Operations including Dividing by a Fraction

Enduring Understandings

Students will understand that ...

- Division and measurement are applied to fractions and decimals as well as to whole numbers.
- Multiplication and division are inverse operations.
- The relationship of the location of the digits and the value of the digits is part of understanding multi-digit operations.
- Division can be represented using multiple formats (manipulatives, diagrams, real-life situations, equations).
- Operations on decimals and whole numbers are based upon place value relationships.
- Problems of area of polygons can be solved by composing and decomposing the polygons.

Essential Question(s):

- How is division related to realistic situations and to other operations?
- What role does place value play in multi-digit operations?
- How can division be represented and interpreted?

Essential Skills:

Students will be able to ...

- Compute quotients of fractions divided by fractions. (6.NS.1)
- Explain the meaning of a quotient determined by division of fractions, using visual fraction models, equations, real-life situations, and language. (6.NS.1)
- Divide multi-digit numbers fluently using the standard algorithm. (6.NS.2)
- Fluently add, subtract, multiply and divide decimals to solve problems. (6.NS.3)
 - Compute with whole numbers. (6.NS.4)

Standards: NS.1, NS.2, NS.3, NS.4

Vocabulary:

reciprocal, inverse operation, multi-digit, greatest common factor, least common multiple, factors, multiples,

prime factorization, compose, decompose, divisor, dividend, quotient, remainder

Instructional Resources:
http://ime.math.arizona.edu/progressions/
http://www.engageny.org/mathematics
http://www.isbe.net/
Eureka Math
NCTM: Illuminations
Achieve the Core Modules
Hunt Institute videos
Tasks:
https://www.georgiastandards.org/Common-
Core/Common%20Core%20framworks/CCGPS Math 6 6thGrade/

TASK: FINDING COMMON FACTORS

Find a solution for the problems below. Draw pictures or use manipulatives to support your solutions

EXPLORATION PROBLEMS:

Two students are having a party. They want to make treat bags for their guests. They want each bag to be identical with nothing leftover. They have 36 Silly Bandz and 72 pieces of bubble gum to put in the bags. What is the greatest number of treat bags they can make and how many of each item will be in each treat bag? Mitzi is making trail mix out of 48 bags of nuts and 32 bags of dried cranberries. She wants each new portion of trail mix to be identical containing the same combination of nuts and cranberries with nothing left over. What is the greatest number of portions of trail mix Mitzi can make and how much of each ingredient will be in each reat bag?

The Junior Beta Club is making food baskets for the local homeless shelter. They asked for donations and they received 88 cans of food and 44 loaves of bread. If they want all the baskets to be the same with nothing left over, how many baskets can they make and how many of each item will be in each basket?

Keesha baked 4 dozen oatmeal cookies and 30 chocolate chip cookies. She wants to divide the cookies into plastic containers with the same amount of cookies in each container. If she wants the container to hold the greatest number of cookies possible how many containers does she need and how many of each cookie will be in each container?

Assessments:

http://www.engageny.org/mathematics

Module 3: NUMBER SYSTEMS: Rational Numbers

Enduring Understandings

Students will understand that ...

• Quantities having more or less than zero are described using positive and negative numbers.

• Number lines are visual models used to represent the density principle: between any two whole numbers are many rational numbers, including decimals and fractions.

• The rational numbers can extend to the left or to the right on the number line, with negative numbers going to the left of zero, and positive numbers going to the right of zero.

• The coordinate plane is a tool for modeling real-world and mathematical situations and for solving problems.

Essential Question(s):

- How are positive and negative numbers used?
- How do rational numbers relate to integers?
- What is modeled on the coordinate plane?

Essential Skills:

Students will be able to ...

- Identify an integer and its opposite and the directions they represent in real-world contexts.(6.NS.5)
- Identify order and absolute value. (6.NS.6c, 6.NS.7a,b,c)
- Use integers to represent quantities in real-world situations (above/ below sea level) (6.NS.5)
- Understand the meaning of 0 and where it fits into a situation. (6.NS.6)
- Represent and explain the value of a rational number as a point on a number line. (6.NS.6)
- Incorporate opposites on the number line or plot opposite points on a coordinate grid where x and y intersect at zero. (6.NS.6b,c, 6.NS.8)
- Represent signs of numbers in ordered pairs as locations in quadrants on the coordinate plane and explain the relationship between the location and the signs. (6.NS.6b,c, 6.NS.8)
- Locate and position integers and other rational numbers on a coordinate plane. (6.NS.6, 6.NS.8)

Standards: NS.5, NS.6, NS.7, NS.8

Vocabulary:

integers, rational numbers, quadrants, absolute value, positive, negative, opposite, coordinate, ordered

pair, y-coordinate, y-coordinate, x-axis, y-axis, origin, distance

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Instructional Resources:	
http://ime.math.arizona.edu/progressions/	
http://www.engageny.org/mathematics	
http://www.isbe.edu/	
Eureka Math	
NCTM: Illuminations	
Achieve the Core Modules	
Hunt Institute videos	
Tasks:	

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx

LEARNING TASK: ABSOLUTE VALUE AND ORDERING

For negative numbers, as the absolute value increases the value of the number decreases. The greater the debt the less money you have. The greater the depth the farther away from the surface.

- **1.** A. A mother dolphin is 150.25 meters below sea level. Her calf is 45 meters below sea level. Which dolphin is farthest from the surface?
 - B. Draw this situation on a vertical number line? Does this illustrate absolute value or ordering (for ordering use <, >)? Justify your answer.
 - C. A mother whale is at 35 meters below the surface and her calf is at the surface. How far does the calf have to swim to get to its mother?
 - D. Draw this situation on a vertical number line? Does this illustrate absolute value or ordering (for ordering use <, >)? Justify your answer.

2. Tonya, Beverly, Ashley, Janet, and Anne want to figure out who has the most debt and who has the most money.

- Tonya -\$5.75 Beverly \$12 Janet \$6.25 Anne -\$4.15 Ashley -\$3 A Graph the five numbers on the number line
- A. Graph the five numbers on the number line.
- B. Put the numbers in order.
- C. What is the absolute value of each term in the list? Graph the absolute value of each term on a horizontal number line.

D. List the absolute values in order

E. Are your orders B and D the same? Why or why not?

3. Christopher said, "I put these in order."
-2.3, 4.75, -8.2, 12, -14.25
Beth disagrees with Christopher. She knows he is wrong. Write a statement that Beth might say to show Christopher his mistake?

4. Michelle's debt is \$25 and Sharif's debt is \$45. Both students think they have the most money. Who is correct? Write a statement to PROVE who has the most.

Assessments:

http://www.engageny.org/mathematics

Module 4: Equations and Expressions

Enduring Understandings:

Students will understand that...

- Properties of operations are used to determine if expressions are equivalent.
- There is a designated sequence to perform operations (Order of Operations).
- Variables can be used as unique unknown values or as quantities that vary.
- Algebraic expressions may be used to represent and generate mathematical problems and real life situations.

Essential Question(s):

- What is equivalence?
- How can properties of operations be used to prove equivalence?
- How are variables defined and used?

Essential Skill:

Students will be able to ...

- Write and evaluate numerical expressions involving whole number exponents. (6.EE.1)
- Read, write, and evaluate expressions in which letters stand for numbers. (6.EE.2a,b,c)
- Understand the relationship of operations and use them to generate equivalent expressions. (6. EE.3)
- Identify when two expressions are equivalent. (6.EE.4)
- Understand that solving an equation is a process which answers a question. (6.EE.5)
- Be able to use substitution to determine whether a given number in a specified set makes an inequality true. (6.EE.5, 6.EE.8)
- Use variables to represent numbers and write expressions to solve real-world problems. (6.EE.6)
- Solve real-world problems using equations with variables where variables are nonnegative rational numbers . (6.EE.7)

Standard: EE.1, EE.2, EE.3, EE.4, EE.5, EE.6, EE.7, EE.8, EE.9

Vocabulary:

evaluate, equivalent, base number, exponent, power, equation, expression, variables, order of operations,

greatest common factor, least common multiple, prime factorization, dividend, quotient, divisor

Instructional Resources:				
http://ime.math.arizona.edu/progressions/				
http://www.engageny.org/mathematics				
http://www.isbe.edu/				
NCTM: Illuminations				
Achieve the Core Modules				
Hunt Institute videos				
Tasks:				

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx

TASK: RULES FOR EXPONENTS

- 1. With a partner determine if the following expressions are equivalent
- a. 22·32–23–1
- b. 22·(32-23)-1
- c. (2·3)2-23-1
- 2. Write an expression of your own using all the operations as well as exponents.
- 3. Rewrite the expression using grouping symbols to give a different answer.

4. Explain the Order of Operations and how it is useful in solving mathematical and real world problems.

TASK: WRITING EXPRESSIONS

1. Within your classroom, have the students find situations where they can role play to compare known and unknown quantities (e.g., Student A (Dory) and Student B (Colleen).For example Dory says, "I have two sisters." Colleen says, "I have Dory – 1 sisters." Dory says, "You have d – 1 sister. You have one sister.") Make sure all operations are included. Write expressions here.

2. Give each pair of students an expression such as x + 957. Challenge them to find a way to evaluate each expression for x = 35, 825, and 373. Then have the students write a real-life context for each expression. Write expressions here.

3. Write each word phrase as an algebraic expression.

6 less than 3t	the product of w and 8
r divided by 15	9 more than twice x
the quotient of 12 and x	the product of x and 6
the sum of three times a and 35	six times the sum of x and 8
a number, x, decreased by 9	a number increased by the quotient of x and 7
15 less than 4 times 11	a number, n, decreased by the difference of x and 7

4. Hannah is 3 years younger than Katie. Joey is twice as old as Hannah.

Let k stand for Katie's age. Write an expression to represent Hannah's age. Using k, write an expression for Joey's age.

Assessments: http://www.engageny.org/mathematics

https://www.georgiastandards.org/Common-Core/Common%20Core%20framworks/CCGPS Math 6 6thGrade/

http://www.isbe.net

Module 5: Geometry: Area, Surface Area, Volume

Enduring Understandings:

Students will understand that...

- Geometry and spatial sense offer ways to envision, to interpret and to reflect on the world around us.
- Area, volume and surface area are measurements that relate to each other and apply to objects and events in our real life experiences.
- Properties of 2-dimensional shapes are used in solving problems involving 3-dimensional shapes.
- The value of numbers and application of properties are used to solve problems about our world

Essential Question(s):

- How does what we measure influence how we measure?
- How can space be defined through numbers and measurement?
- How does investigating figures help us build our understanding of mathematics?
- What is the relationship between 2-dimensional shapes, 3-dimensional shapes and our world?

Essential Skills:

Students will be able to ...

- Divide irregular shapes into triangles and rectangles in order to determine area. (6.G.1)
- Draw polygons and determine distance (length) between two vertices when given coordinates. (6.G.3)
- Calculate the volume of a right rectangular. (6.G.2)
- Apply the formula to solve real world mathematical problems involving volume with fractional edges/ lengths. (6.G.2)
- Represent 3D figures using nets of triangles and rectangles. (6.G.4)
- Solve real world problems involving surface areas using nets. (6.G.4)

Standards: G.1, G.2, G.3, G.4, EE.2, EE.5, EE.6, EE.7

Vocabulary:

area, surface area, volume, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right

triangle, quadrilateral, rectangles, squares, parallelograms, trapezoids, rhombi, kites, right rectangular

prism, diagonal

Instructional Resources:

http://ime.math.arizona.edu/progressions/

http://www.engageny.org/mathematics

http://www.isbe.edu/

Eureka Math

NCTM: Illuminations

Newspaper and magazine graphs

Achieve the Core Modules

Hunt Institute videos

Tasks:

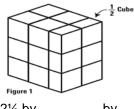
Block Part-y

The cube shown to the right represents the unit cube. It has the dimensions 1 by 1 by 1. Color the faces of the unit cube *blue*.

What is the volume of the cube?

1	х	1	х	1	=	cubic	unit
	~		~		_	 oubio	unit

The rectangular prism in *Figure 1* is made up of some unit cubes as well as other cubes that have been cut in half. What are the dimensions of Figure 1?



2½ by _____ by _____

Color the faces of the unit cubes blue.

Color the faces of the $\frac{1}{2}$ cubes green.

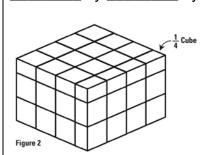
How many uncut (unit) cubes are in the figure?

How many 1/2 cubes are in the figure?

Without using the formula for finding volume, explain how you could find the volume of the prism.

Now show how to find the volume using the formula.

The rectangular prism in *Figure 2* is made up of some unit cubes, some $\frac{1}{2}$ cubes and some $\frac{1}{4}$ cubes. What are the dimensions of Figure 2?



Color the faces of the unit cubes *blue*.

Color the faces of the 1/2 cubes green.

Color the faces of the 1/4 cubes red.

How many unit cubes are in the figure? _____

How many ½ cubes are in the figure?

How many ¼ cubes are in the figure? _____

Without using the formula for finding volume, explain how you could find the volume of the prism.

Now show how to find the volume using the formula.

Assessments:

http://www.engageny.org/mathematics

Module 6: Statistics and Probability

Enduring Understandings:

Students will understand that ...

- Statistical questions and the answers account for variability in the data.
- The distribution of a data set is described by its center, spread, and overall shape.
- Measures of center for a numerical set of data are summaries of the values using a single number.
- Measures of variability describe the variation of the values in the data set using a single number.

Essential Question(s):

- How do we analyze and interpret data sets?
- When is one data display better than another? How do mathematicians choose to display data in strategic ways?
- When is one statistical measure better than another?
- What makes a good statistical question?

Essential Skills:

Students will be able to ...

- Identify statistical questions. (6.SP.1)
- Represent a set of data collected to answer a statistical question and describe it by its center, spread, and overall shape. (6.SP.2)
- Represent and explain the difference between measures of center and measures of variability. (6.SP.2, 6.SP.3)
- Display numerical data in plots on a number line, histogram, and box plots. (6.SP.4)
- Use language to summarize numerical data sets in relation to their context.(6.SP.5)
- Give quantitative measures of center and variability as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (6.SP.5)
- Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (6.SP.5)

Standards: SP.1, SP.2, SP.3, SP.4, SP.5

Vocabulary:

data, statistics, variability, center, spread, mean, mode, range, quartile, ox plot, distribution, histogram,data

set, distribution, sample size, deviation

Instructional Resources:				
http://ime.math.arizona.edu/progressions/				
http://www.engageny.org/mathematics				
http://www.isbe.net/				
Eureka Math				
NCTM: Illuminations				
Newspaper and magazine graphs				
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Tasks:				

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx

http://www.isbe.net

TASK: LEARNING ABOUT BOX PLOTS KINESTHETICALLY-HOW LONG IS A MINUTE?

Do you think you can determine how long a minute is without looking at a clock? With your partner and your stop watch, you will each attempt to determine when you think a minute has passed without looking at a watch or clock. All time will be recorded in SECONDS – do not convert to minutes. You will take turns measuring and timing.

1. Within your pair decide who will be the "timer" and who will be the "guesser." The "timer"

will have the stop watch and direct the "guesser" when to start. The "guesser" will attempt to tell how long a minute is without looking at a clock.

2. When instructed by teacher, the "timer" tells the "guesser" to begin. When the "guesser" believes a minute has passed, he should say, "stop" quietly. Timer - record the time that has passed to the nearest second. Do not tell your partner how much time actually passed! The "timer" needs to record the "guesser" time down on a sheet of paper.

3. Switch roles and repeat #2.

4. Share times with your partner. Write **your own** time down (in seconds) on a sheet of paper, large enough so that everyone can see it (one sheet per student).

As a class, find the median of the data by counting to the middle. Have a student put a sticky note with "Q2" where the median is. Note to class - The median is also known as Quartile 2 (Q2)

As a class, find the median of the lower half of numbers. Have a student put a sticky note with "Q1" on this place. Note to class - This is Quartile 1 (Q1), which is the median of the lower half of the data.

As a class, find the median of the upper half of numbers. Have a student put a sticky note with "Q3" on this place. Note to class - This is Quartile 3 (Q3), which is the median of the upper half of the data.

Create a human box plot or box plot using class data however the class decides. (Sticky notes and white board or using string for human box plot.)

From the data draw a box plot of the class's data that is the same box plot that you made with your bodies. Remember to draw the number line FIRST.

5. How many students participated in the live Box Plot? _____

6. How many students were INSIDE the box? _____ What percent is this? _____

7. Where are the other students?

8. What conclusions can you make about the each quartile?

Assessments:

http://www.engageny.org/mathematics